Chapter 6 – Day 3

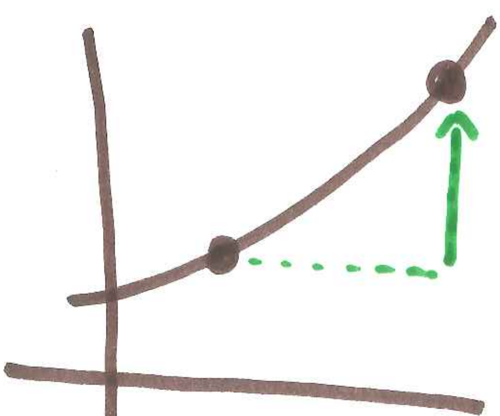
Increasing and Decreasing Functions

f is <u>increasing</u> on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

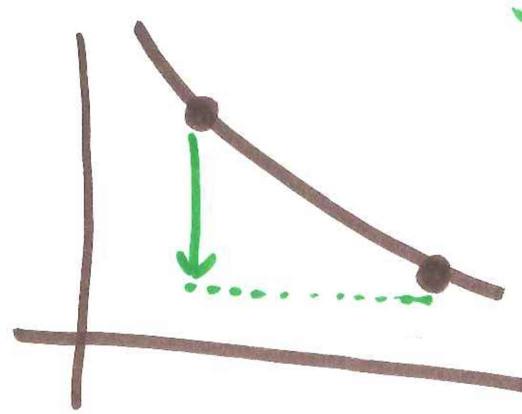
"rising"

f is <u>decreasing</u> on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.

"falling"

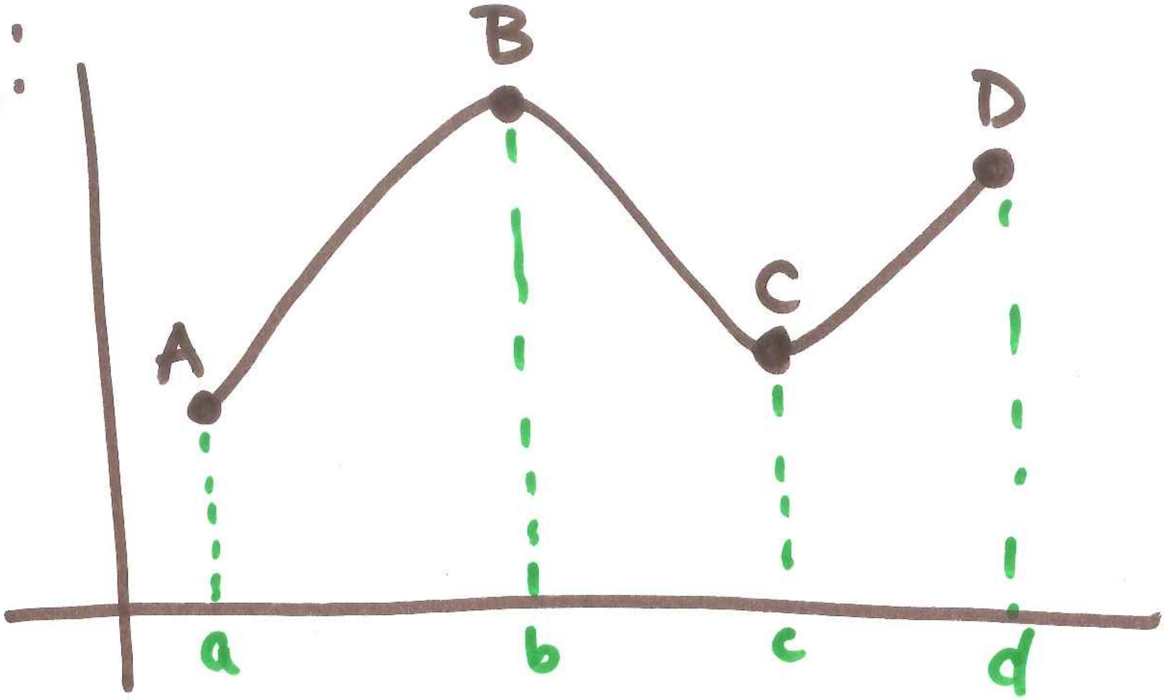


f increasing



f decreasing

Ex:



Where is f increasing and decreasing?

f increasing on $[a,b]$ and $[c,d]$

f decreasing on $[b,c]$

Where on this graph is the slope of the tangent line positive? (a,b) and (c,d)

Where negative? (b,c)

Ex: Suppose $f(3)=7$ and $f(5)=12$. f is increasing on $(3,5)$ and decreasing on $(-\infty,3)\cup(5,\infty)$. Are the following possible?

a) $f(1)=3$ not possible

b) $f(1)=10$ possible

c) $f(4)=5$ not possible

d) $f(6)=10$ and $f(8)=15$ not possible

e) $f(6)=10$ and $f(8)=6$ possible

The previous example tells us that:

- if $f(x)$ is increasing then $f'(x)>0$
- if $f(x)$ is decreasing then $f'(x)<0$

Mean Value Theorem: if f is continuous on [a, b] and differentiable at every point between a and b, then there exists some point $x = c$ between a and b such that

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

ARoC from a to b = IRoC at c

Ex: let $Q(t) = t^2$. Find a value $A \neq 1$ such that the average rate of change of $Q(t)$ from 1 to A equals the instantaneous rate of change of $Q(t)$ at $t=3$.

$$\frac{Q(A) - Q(1)}{A-1} = Q'(3)$$

$\leftarrow Q'(t) = 2t$

$$\frac{A^2 - 1^2}{A-1} = 2(3)$$

$$\frac{A^2 - 1}{A-1} = 6$$

$$\frac{\cancel{(A-1)}(A+1)}{\cancel{A-1}} = 6$$

$$A+1 = 6$$

$$A = 5$$

Ex: let $f(x) = x^3 - x$ on the interval $[-1, 3]$. Find all numbers c that satisfy the MVT.

$$\frac{f(3) - f(-1)}{3 - (-1)} = f'(c)$$

$$\frac{(3^3 - 3) - ((-1)^3 - (-1))}{4} = (3x^2 - 1)\Big|_c$$

$$\frac{24}{4} = 3c^2 - 1$$

$$6 = 3c^2 - 1$$

$$7 = 3c^2$$

$$\frac{7}{3} = c^2$$

$$c = \pm\sqrt{\frac{7}{3}}$$

$-\sqrt{7/3}$ not in interval $[-1, 3]$

$$\boxed{c = \sqrt{\frac{7}{3}}}$$